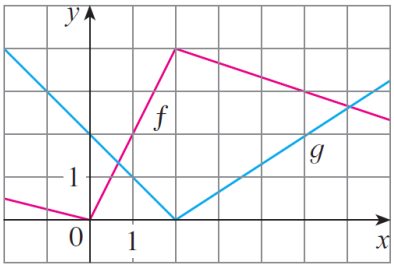
Exercise 49

If f and g are the functions whose graphs are shown, let $u(x) = f(x)g(x)$ and $v(x) = f(x)/g(x)$.

(a) Find $u'(1)$. (b) Find $v'(5)$.



Solution

Evaluate the derivative of $u(x)$ using the product rule.

$$u'(x) = f'(x)g(x) + f(x)g'(x)$$

Evaluate the derivative of $v(x)$ using the quotient rule.

$$v'(x) = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$$

At $x = 1$, the slope of f is 2 and the slope of g is -1: $f'(1) = 2$ and $g'(1) = -1$. Use this information to evaluate $u'(1)$.

$$u'(1) = f'(1)g(1) + f(1)g'(1) = (2)(1) + (2)(-1) = 0$$

At $x = 5$, the slope of f is $-1/3$ and the slope of g is $2/3$: $f'(5) = -1/3$ and $g'(5) = 2/3$. Use this information to evaluate $v'(5)$.

$$v'(5) = \frac{f'(5)g(5) - g'(5)f(5)}{[g(5)]^2} = \frac{\left(-\frac{1}{3}\right)(2) - \left(\frac{2}{3}\right)(3)}{2^2} = -\frac{2}{3}$$